

Quantum statistics in time-modulated exciton-photon system

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We consider a system consisting of a large individual quantum dot with excitonic resonance coupled to a single mode photonic cavity in the nonlinear regime when exciton- exciton interaction becomes important. We show that in the presence of time-modulated external coherent pumping the system demonstrates essentially non classical behavior reflected in sub-Poissonian statistics of exciton- and photon-modes and the Wigner functions with negative values in phase-space for time-intervals exceeding the characteristic time of dissipative processes, $t \gg \gamma^{-1}$. It is shown that these results are cardinally different from the analogous results in the regime of the monomode continuous-wave (cw) excitation.

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I. INTRODUCTION

When light interacts with matter one should discriminate between two qualitatively different regimes, namely weak and strong coupling. For the first of them the spectrum of the material system remains unchanged, and light- matter interaction results in single acts of the emission and absorption of the photons. On the contrary, in the regime of strong coupling light and matter can not be anymore treated independently from each other, and hybrid half- light half matter modes appear in the system. Those modes are known as polaritons. Nowadays the variety of polaritonic systems is impressive and include such widely studied cases as surface plasmon polaritons [1] and cavity polaritons [2]. In the latter case photonic mode is coupled with excitonic transition in the quantum well, quantum wire or individual quantum dot (QD).

Coupling between QD and photonic mode of zero dimensional cavity lies in heart of such rapidly developing branch of science as cavity quantum electrodynamics (cQED). The problem is important not only because of the fundamental aspects brought forward by the interaction of material systems with photons [3], but also because of the potential application of cQED to quantum information processing [4–6]. From the point of view of experimental realization, excitons in individual QD can be brought to strong coupling with confined electromagnetic mode provided by a pillar (etched planar cavity) [7], the defect of a photonic crystal [8] or the whispering gallery mode of a microdisk [9, 10], among others. In the regime of weak excitation such structures have demonstrated the Rabi doublet in their optical spectra, which is characteristic of the mode anticrossing that marks the overcome of dissipation by the coherent exciton-photon interaction.

These achievements open the way to new research area, namely investigation of the pure quantum effects originating from strong exciton-photon coupling[11]. Although the system exhibits strong coupling, it is usually

not known in which quantum state it is actually realized. To be useful for quantum information applications, one should be able to manipulate not just mean number of the particles in the system, but have a tool to monitor and control their statistics. In this context, the possibility to create the states different from essentially classical coherent and thermal states is highly desirable. If the energy of the system scales linearly with the number of particles, it is essentially classical, [12] and adding or removing a single particle will not change its behaviour. Therefore, the analysis of the nonlinear effects is highly desirable.

With QDs in microcavities, two types of strong nonlinearities are expected, both associated with the excitons [13]. The first one comes from Pauli exclusion, that arises from the fermionic character of the particles forming an exciton. It becomes extremely important for the case of the small size QDs where, similar to individual atoms, the excitation of more than one exciton becomes impossible. Pauli blocking leads to the radical transformation of the spectrum of the system, which changes from the Rabi doublet to Mollow triplet when the intensity of the external pump is increased [14–18].

The second is Coulomb repulsion between the excitons, again arising from their composite nature. This mechanism is important for the case of the excitons in the QWs and large QDs. In the former case it leads to the blueshift of the polaritonic modes increasing with the intensity of the external pump- the effect which can be satisfactory described within the frameworks of the mean-field approximation. The case of an individual large QD inside the cavity is, however, more tricky. Coulomb repulsion between the excitons in this configuration leads to the emergence of rich multiplet structure in the emission spectrum [19], which reveals the pure quantum nature of light- matter coupling.

In the present paper we analyze further the essentially quantum effects arising from nonlinearities in coupled QD- cavity system. The novelty of our scheme stems

from the idea that quantum systems can display qualitatively new forms of behavior when driven by fast time-periodic modulations. Particularly, the application of a sequence of tailored pulses as well as time-modulated cw field leads to improving the degree of quantum effects in open cavity nonlinear systems and onset of qualitatively new quantum effects. This approach was recently exploited for the formation of a high degree continuous-variable entanglement in the nondegenerate optical parametric oscillator [20, 21], for generation of Fock states in Kerr nonlinear resonator (KNR) driven by a sequence of Gaussian pulses [22, 23] and for demonstration of multi-photon blockades in pulsed regimes of dissipative KNR beyond stationary limits [24, 25]. It has also demonstrated that amplitude modulation can improve the performance of single photon sources based on quantum dot [26]. The idea to enrich quantum physical systems by designing a time modulation has been explored in several other fields of research including periodically driven nonlinear oscillator [27] and periodically driven quantum matter [28].

Here we focus on consideration of cavity modes in the regimes of strong exciton-photon coupling and strong exciton-exciton interaction with respect to the rates of damping of the photonic- and exciton-modes. In these regimes the transition frequencies between energy levels of the systems without any interaction with external field are differently spaced in the quantum regime. Thus, we enable spectroscopic identification and selective excitation of transitions between combined exciton-photon number states. This consideration provides the framework of master equation and the numerical method of quantum trajectories on the base of excitation numbers, Q Mandel parameter or the second-order correlation function. We focus also on the Wigner functions of exciton and photonic modes that allow phase-space monitoring exciton-photon coupling in quantum treatment.

We show that in the presence of time-modulation the ensemble-averaged mean photon numbers, the populations of photon-number and exciton-number states, and the Wigner functions are nonstationary and exhibit a periodic time dependent behavior, i.e. repeat the periodicity of the pump laser in an over transient regime. We demonstrate the possibility of the observation of purely quantum effects, namely sub-Poissonian statistics of exciton- and photon-modes and the Wigner functions with negative values in phase-space for time-intervals exceeding the characteristic time of dissipative processes, $t \gg \gamma^{-1}$. It is shown that these results are cardinally different from the analogous results in cw regime of excitation. Beside this we investigate temperature noisy effects for a cavity at finite temperatures. It leads to applications in simulating of more realistic exciton-photon systems as well as to study of unusual quantum phenomena connecting quantum engineering and temperature.

The paper is organized as follows. In Sec. II, we present the effective Hamiltonian for the periodically driven polariton system and describe the physical quantities of interest.

In Sec. III we study time-evolution of the mean photon numbers as well as the exciton numbers, quantum statistics of excitonic and photonic modes on base of the Q Mandel parameter and the second-order correlation function for zero-delay time. We analyze also the distributions of photon-number states and the phase-space properties of photon-and exciton-modes on base of the Wigner functions. The effects coming from a thermal reservoir is also briefly analysed. We summarize our results in Sec. IV.

II. COUPLED EXCITON-PHOTON SYSTEM

The system consists of coupled fundamental photonic dot and exciton modes driven by cw field with mean frequency ω and time-modulated amplitude. The Hamiltonian of driven photon-exciton system in the rotating wave approximation (RWA) reads:

$$H = \Delta_{ph} a^\dagger a + \Delta_{ex} b^\dagger b + \chi b^{\dagger 2} b^2 + g(ba^\dagger + b^\dagger a) + (\Omega_1 + \Omega_2 \exp(-i\delta t))a + H.c, \quad (1)$$

where a^\dagger, a are creation and annihilation operators of the photon mode, b^\dagger, b are creation and annihilation operators for the exciton mode, g is the exciton-photon coupling constant, χ is the strength of exciton-exciton interaction, $\Delta_{ph} = \omega_{ph} - \omega$, $\Delta_{ex} = \omega_{ex} - \omega$ are detunings between mean frequency of the driving field and the frequencies of the photonic and exciton modes. Ω_1 and Ω_2 are the components of complex amplitude of the driven field and δ is the frequency of the modulation. The case $\Omega_2 = 0$ describes the exciton-photon cavity driven by cw monochromatic field treated within RWA. Such situation can be realized also if photon-exciton system is driven by two fields with different frequencies. In this case, the Hamiltonian of the system in RWA is reduced to Eq. 1 with δ being the difference between frequencies of the driving fields.

In realistic systems one should necessarily take into account the dissipation, because the modes suffer from losses due to partial transmission of light through the mirrors of the photonic cavity, non-radiative decay of excitons and decoherence. We consider these effects by assuming that the interaction of driven photon-exciton system with heat reservoir gives rise to the damping rates of modes γ_a and γ_b . We trace out the reservoir degrees of freedom in the Born-Markov limit assuming that system and environment are uncorrelated at initial time $t = 0$. This procedure leads to the master equation for the reduced density matrix in the Lindblad form. The master equation within the framework of the rotating-wave approximation, in the interaction picture corresponding to the transformation $\rho \rightarrow e^{-i\omega a^\dagger a t} \rho e^{i\omega a^\dagger a t}$ reads as

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_{i=1,2,3,4} \left(L_i \rho L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho - \frac{1}{2} \rho L_i^\dagger L_i \right), \quad (2)$$

where $L_1 = \sqrt{(n_{th} + 1)\gamma}a$, $L_2 = \sqrt{n_{th}\gamma}a^\dagger$, $L_3 = \sqrt{(n_{th} + 1)\gamma}b$ and $L_4 = \sqrt{n_{th}\gamma}b^\dagger$ are the Lindblad operators, γ is a dissipation rate, and n_{th} denotes the mean number of quanta of a heat bath. Here, for simplicity we assume that the decay rates and the mean numbers of quanta of a heat bath are equal for the both modes, $\gamma_a = \gamma_b = \gamma$.

We analyze the mean values of excitation numbers as well as we probe the strength of quantum fluctuations of exciton and photonic modes via the Mandel factor and the normally ordered second-order correlation functions of the photon numbers and excitation numbers. Beside this we monitor phase properties of both modes by the Wigner function. These quantities of interest are calculated by using the reduced density operators of the photons $\rho_a(t)$ and of the excitons $\rho_b(t)$. These operators are constructed from the full density operator of the system $\rho(t)$ by tracing out the excitonic or photonic modes respectively,

$$\rho_a(t) = Tr_b(\rho), \quad (3)$$

$$\rho_b(t) = Tr_a(\rho). \quad (4)$$

For the system under time modulated external pumping the ensemble-averaged mean oscillatory excitation numbers as well as the other physical quantities exhibit a periodic time-dependent behavior after initial transient time interval. Using the master equation above we calculate the time-evolution of the Q Mandel factor for the photonic and exciton modes. For photonic mode it is defined as

$$Q(t) = \frac{\langle (\Delta n(t))^2 \rangle - \langle n(t) \rangle}{\langle n(t) \rangle} \quad (5)$$

where $\langle (\Delta n(t))^2 \rangle = \langle (a^\dagger a)^2 \rangle - (\langle a^\dagger a \rangle)^2$ describes the deviation of the excitation number uncertainty from the Poissonian variance, $\langle (\Delta n)^2 \rangle = \langle n \rangle$. The case $Q = 0$ corresponds to Poissonian statistics. If $Q > 0$, the statistics is super-Poissonian, if $Q < 0$ it is sub-Poissonian, and analogous for the exciton mode.

The Mandel factor is connected with the normalized second-order correlation function for zero delay time $g^{(2)}$ defined (for the photonic mode) as:

$$g^{(2)}(t) = \frac{\langle a^\dagger(t) a^\dagger(t) a(t) a(t) \rangle}{(\langle a^\dagger(t) a(t) \rangle)^2}. \quad (6)$$

For a short counting time-intervals the approximate relation between these quantities reads as $\langle (\Delta n)^2 \rangle = \langle n \rangle + \langle n \rangle^2 (g^{(2)} - 1)$. Thus, the condition $g^{(2)} < 1$ corresponds to the sub-Poissonian statistics, $\langle (\Delta n)^2 \rangle < \langle n \rangle$.

We analyze the master equation numerically using well known quantum state diffusion method [29]. According

to this method, the reduced density operator is calculated as the ensemble mean over the stochastic states describing evolution along a quantum trajectory.

It should be mentioned that exciton-photon system is actually presented as the model of driving coupled oscillators with anharmonic term. Such model has been focus of considerable attention and this interest is justified by many applications in different contexts. Particularly, in field of quantum devices for a few-photon level one application concerns to realization of strong photon blockade in photonic molecules with modest Kerr-nonlinearity of the photon using two coupled photonic cavities [30–32].

III. IMPROVEMENT OF QUANTUM EFFECTS BY PERIODIC TIME-MODULATION

In our further consideration we concentrate on analysis of coupled exciton-photon in quantum regimes at low level of quanta. This analysis is performed by using the following set of dimensionless parameters Δ/γ , χ/γ , g/γ , as well as parameters of amplitude modulation Ω_1/γ , Ω_2/γ and the frequency of modulation δ .

Let us discuss the operational regimes in more details. If energy levels of the coupled exciton-photon states are well resolved we can consider near to the resonant selective transitions between lower photon-exciton states. In this way, the detunings play an important role when identifying the spectral lines, thus we arrange the detunings of modes to reach a qualitative quantum effects. We estimate the detuning by using the results on the structure of energy levels of a quantum dot in a microcavity in the nonlinear regimes Ref. [19]. In this paper the optical spectrum at resonance transition was studied by diagonalizing the Hamiltonian 1 without the driving term. In this case the total number of excitations is conserved, and eigenstates of the total particle number $a^\dagger a + b^\dagger b$ with eigenvalue m can be used as the m th manifold of the system. Particularly, two-photon excitation of the system with $m = 2$ leads to the eigenvectors that involves triplet states $|2, 0\rangle$, $|1, 1\rangle$, $|0, 2\rangle$ with photonic and exciton number states. If we neglect the exciton-exciton interaction, the energy levels form the Rabi triplet $E_+ = 2\omega_{ph} + 2g$, $E_0 = 2\omega_{ph}$, $E_- = 2\omega_{ph} - 2g$. In this way, two-photon resonant frequencies leading to excitation of the level E_- equals to $2\omega_2 = E_- - E_g$, where E_g is the level of ground state. Therefore, $\omega_2 = \omega_{ph} - g$ and hence the detunings $\Delta_{ex} = \omega_{ex} - \omega_2$, $\Delta_{ph} = \omega_{ph} - \omega_2$ are $\Delta_{ph} = \Delta_{ex} = g$. In this approach the effects of exciton-exciton interaction can be included by using numerical calculations Ref. [19]. For strong driving field the detunings can be also shifted due to Stark effects. Thus, for the concrete calculations, later we use the approximate value of detunings corresponding to the parameters g/γ and χ/γ . Note, that such analysis seems to be rather qualitative than quantitative but allows us to estimate the values of detunings. Indeed, considering strong analysis of the selective excitation of cavity modes we need in calculations of the other

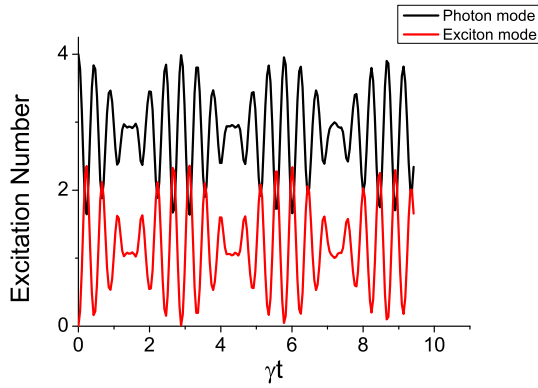


FIG. 1: Time-evolution of the mean excitation numbers of photonic and excitonic modes for the nondissipative case, without any driving. The parameters are as follows: $\Delta_{ph}/\gamma = \Delta_{ex}/\gamma = 7.12$, $g/\gamma = 5$, $\chi/\gamma = 4$.

resonant frequencies corresponding the large numbers of quanta, $m > 2$. Below, for concrete calculations we use the values $\Delta_{ph}/\gamma = \Delta_{ex}/\gamma = 7.12$ that are estimated for the parameter $g/\gamma = 7$.

For the illustrative purposes let us first consider the case of the non-dissipative system. The typical results for the time-evolution of the mean excitation numbers of both photon-mode and exciton-mode due to the coupling between modes and the exciton-exciton nonlinearity are shown at the Figs. 1. We assume that at $t = 0$ the exciton mode is in vacuum state $\langle n(0) \rangle_b = 0$ while the four-photon number state is injected into the resonator, $\langle n(0) \rangle_a = 4$. As we see, without any driving the total number of quanta is conserved and the dynamics of excitation numbers display oscillatory behaviour, where are observed the Rabi transitions between the modes with collapses and revivals effects.

The collapses and revivals are well known phenomena in quantum optics, particularly, in the context of the Jaynes-Cummings model (see, for example [33]) and for ion-trap system [34], [35]. Here, we obtain interesting collapses and revival effects in the simplest model as a manifestation of nonlinear exciton-exciton interaction that leads to spaced energies of exciton number states. Note, also that in this case conservation of total particle numbers lead to anti-phase dynamics of mode that is violated in the presence of driving.

Now we turn to the realistic case where dissipation and decoherence are taken into account considering pure quantum effects for zero temperature cavity. We start from the case of $\Omega_2 = 0$, which corresponds to the single mode cw excitation, and the initial state corresponding to no excitations in the cavity. The typical results for the excitation numbers, the Mandel parameters and the contour plots of the Wigner functions of two modes are depicted in Fig. 2. As we see, the mean excitation numbers and the Mandel parameter show Rabi-like oscillations for short time-intervals in the transient regime and reach the

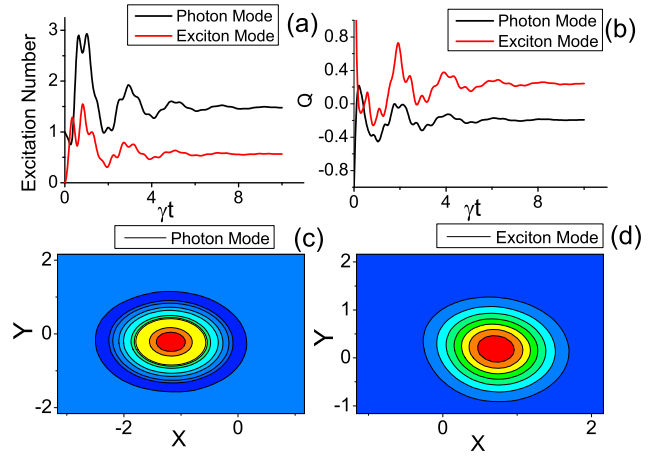


FIG. 2: The results for the case of single mode driving, $\Omega_2 = 0$. (a) temporal dependence of the mean excitation numbers of modes; (b) temporal dependence of the Mandel parameters for excitonic and photonic modes. The contour plots of the Wigner functions of photonic mode (c) and excitonic mode (d) for time intervals corresponding to the maximal values of excitation numbers exceeding the characteristic decoherence time. The parameters are as follows: $\Delta_{ph}/\gamma = 7.12$, $\Delta_{ex}/\gamma = 7.12$, $g/\gamma = 5$, $\chi/\gamma = 1$, $\Omega_1/\gamma = 5$, $\Omega_2/\gamma = 0$, $\delta/\gamma = 0$.

equilibrium in the steady-state regime. From Fig. 2(a) we conclude that for used parameters the system is operated in strong quantum regime at level of small excitation numbers. In stationary, over-transition regime the exciton Mandel parameter $Q_b > 0$, while $Q_a = -0.2$ in steady state limit, i.e. photonic mode displays sub-Poissonian statistics. The Wigner functions of both mode are positive in all phase space but contours of the Wigner function for photonic mode have slightly squeezed form corresponding to small sub-Poissonian statistics.

The typical results for the case of time-modulated excitation with $\Omega_2 \neq 0$ are presented in the Figs. 3, 4 and 6 for various parameters of the exciton-photon system. As one can see at the Figs. 3(a) and 3 (b), for the coupling constants: $\chi/\gamma = 1$ and $g/\gamma = 5$, the mean excitation numbers and the Mandel parameters repeat the periodicity of the pump laser in an over transient regime. It is remarkable that time-modulation of modes leads to formation of highly sub-Poissonian statistics compared to the case of monochromatic driving (see, results depicted in Fig. 2). Indeed, in this case for the photonic mode the values of Mandel parameter can reach the values of $Q_a \approx -0.65$ and for the exciton mode $Q_b \approx -0.45$. If one compare this numbers with those obtained earlier for a single mode cw pumping, one can make a conclusion that quantum effects in the behavior of exciton-photon system becomes more pronounced for the case of the time-modulated pumping. Note, that most negative values of the Mandel parameters are realized for the time-intervals corresponding to the maximal values of the occupations of the modes. The quantum effects are thus more essential for the comparatively large number of the quanta. The

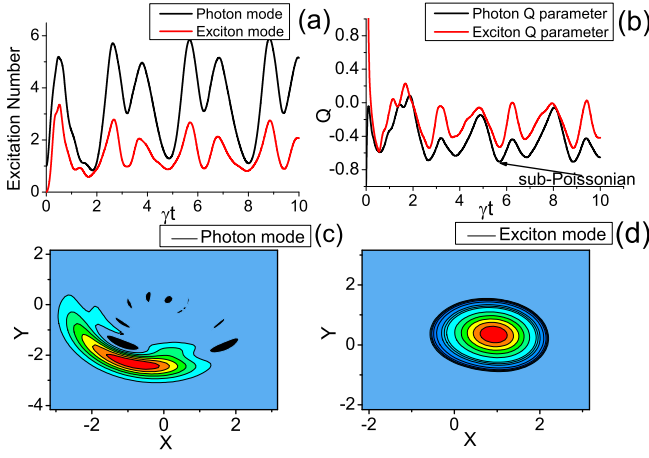


FIG. 3: The results for the case of periodically driven photon-exciton system. (a) temporal dependence of the mean excitation numbers for the excitonic and photonic modes; (b) temporal dependence of the Mandel parameters for both modes; (c) the Wigner function of the photonic mode corresponding to the maximal occupancy of the photonic mode; (d) the Wigner function of the photonic mode corresponding to the minimal occupancy of the photonic mode. The parameters are as follows: $\Delta_{ph}/\gamma = 7.12$, $\Delta_{ex}/\gamma = 7.12$, $g/\gamma = 5$, $\chi/\gamma = 1$, $\Omega_1/\gamma = 5$, $\Omega_2/\gamma = 5$, $\delta/\gamma = 2$.

contour plots of the Wigner functions for maximal and minimum values of photonic mode are presented in Figs. 3(c),(d), respectively. Note, that for the parameters considered when the photon excitation number corresponds to its maximum, the corresponding Wigner function has a region in phase-space where it is negatively defined (these regions are shown in black). On the contrary, for the times corresponding to the minimal photon occupancies the Wigner function is almost Gaussian and Mandel parameters are very close to zero. This indicates that in these moments the state of the system is very close to coherent and its behavior is essentially classical.

Increasing of the coupling constant of the exciton-photon coupling and the strength of exciton-exciton nonlinear interaction makes quantum effects more pronounced, as it is shown in Fig. 4 for both excitonic and photonic modes and the parameters: $\chi/\gamma = 3$, $g/\gamma = 7$. Note, that for these parameters the detunings $\Delta_{ph}/\gamma = 7.12$, $\Delta_{ex}/\gamma = 7.12$ approximately correspond to two-photon selective excitation of the level E_- from vacuum state as it has been shown above.

The time-evolution of averaged excitation numbers are depicted in Fig. 4(a). Comparing these results with analogous ones shown in Fig. 3(a) we conclude, that increasing of the parameters χ/γ and g/γ and leaving the other parameters without change, leads to increasing of the level of photon excitation numbers and decreasing the excitation numbers of exciton mode. Considering quantum statistics of modes (see, Fig. 4(b)) we conclude that this regime displays a deeply sub-Poissonian statistics with Mandel parameter achieving the values of $Q_a \approx -0.8$

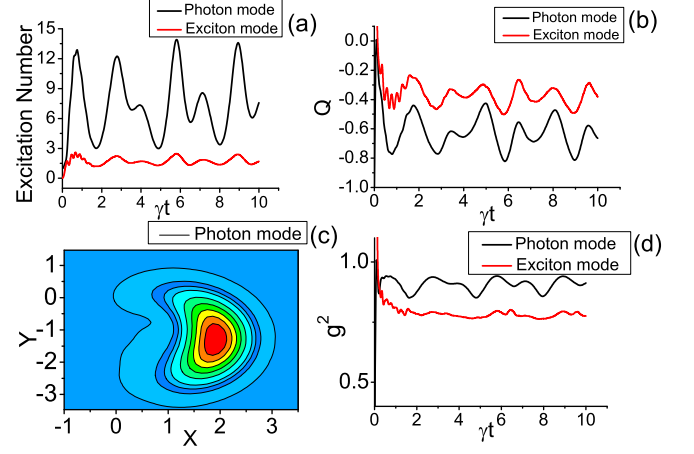


FIG. 4: The results for the case of periodically driven photon-exciton system in deeply sub-Poissonian regime. (a) time-dependence of the mean excitation numbers; (b) time-dependence of the Mandel parameters for excitonic and photonic modes; (c) the contour plots of the Wigner function corresponding to the maximal occupancy of the photonic mode; (d) time-dependent normalized second-order correlation functions for photonic- and exciton-modes. The parameters are as follows: $\Delta_{ph}/\gamma = 7.12$, $\Delta_{ex}/\gamma = 7.12$, $\chi/\gamma = 3$, $g/\gamma = 7$, $\Omega_1/\gamma = 5$, $\Omega_2/\gamma = 5$, $\delta/\gamma = 2$.

for photonic mode and $Q_b \approx -0.5$ for excitonic mode in their minima. It is also interesting to present results on quantum statistics of exciton-photon system within the framework of the normalized second-order correlation function by using the above formulas. The results for nonstationary correlation functions for photonic and excitonic modes versus dimensionless time intervals are depicted in Fig. 4(d). As we see, in this regime the results display photon anti-bunching as well as the exciton anti-bunching for all time-intervals. However, the anti-bunching for exciton mode is slightly stronger than for the photonic mode. Beside this, the correlation function of exciton-mode shows monotonous time-behaviour, with $g^{(2)} \approx 0.76$, while analogous result for photonic mode is non-monotonous with $g^{(2)} \approx 0.82$ at its minima at definite time intervals corresponding to the minimal values of the mean photon number. Note, for the comparison that for $|2\rangle$ pure Fock state the normalized second-order photon correlation function equals to 0.5. Thus, the obtained results reflect excitations of states with high-order $m > 2$ th manifolds and probably describe nonclassically of photon-exciton states for moderate number of quanta. A more complete description of the system can be obtained by calculation of the Wigner function in phase space. In this way, the contour plots of the Wigner function for photonic mode depicted in Fig. 4(c) displays the typical form corresponding to light with sub-Poissonian statistics.

Additionally, the probability distributions of photon-number and exciton-number states for the definite time-intervals are presented in Fig. 5. These results shows

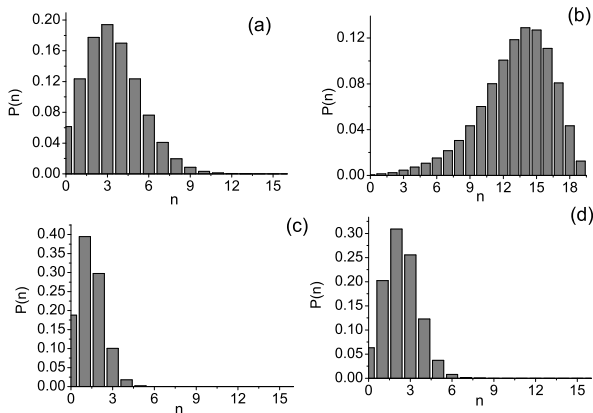


FIG. 5: The photon number distributions at the minimum level of quanta (a) and at the maximum level of quanta (b). The exciton number distributions at the minimum level of quanta (c) and at the maximum level of quanta (d). The parameters are as follows: $\Delta_{ph}/\gamma = 7.12$, $\Delta_{ex}/\gamma = 7.12$, $\chi/\gamma = 3$, $g/\gamma = 7$, $\Omega_1/\gamma = 5$, $\Omega_2/\gamma = 5$, $\delta/\gamma = 2$.

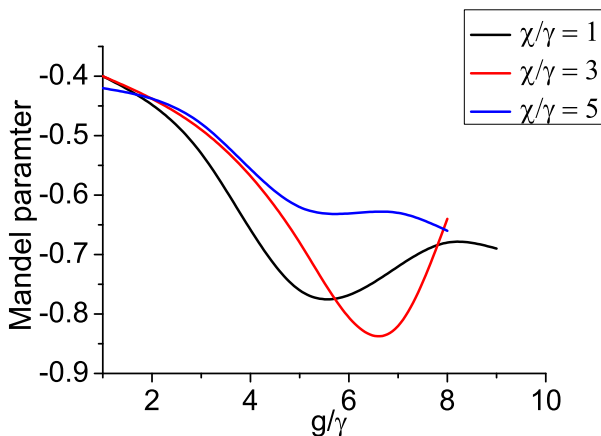


FIG. 6: The dependence of the minimal value of the Mandel parameter on the exciton-photon coupling strength for several values of the exciton-exciton interaction: (a) $\chi/\gamma = 1$; (b) $\chi/\gamma = 3$; (c) $\chi/\gamma = 5$. The other parameters are taken as: $\Delta_{ph}/\gamma = 7.12$, $\Delta_{ex}/\gamma = 7.12$, $\Omega_1/\gamma = 5$, $\Omega_2/\gamma = 5$, $\delta/\gamma = 2$.

that non-selective excitations of both photonic and excitonic modes are realized for the parameters used. Indeed, the distributions of photon numbers are relatively large and centred around the maximal and minimal values of mean photon numbers (see Figs. 4 (a), (b)). Nevertheless, the distributions considerably differ from the Poissonian distribution. The distributions of exciton numbers correspond to excitations of mode at level of a few quanta in accordance with the results of mean exciton numbers.

It is interesting to analyse the minimal values of the Mandel factor in its time-evolution for various coupling constants of the exciton-photon coupling and the strengths of exciton-exciton nonlinear interaction. The

results for photonic mode in dependence from the coupling constant g/γ are presented in Fig. 6 for three values of the parameter χ/γ and the fixed value of the amplitude of driving field. One can clearly see that for the definite parameters of driving time-dependent field, exciton-exciton interaction and the detunings there exist an optimal value of the ratio g/γ where the quantum effects in the behavior of the system becomes most pronounced. These results indicate the regime presented in Fig. 4 as most preferable for production of strong sub-Poissonian statistics. It should be mentioned that the parameters of the Gaussian pulses in above consideration are free parameters and they might be chosen in order to find optimal regimes producing high degree of sub-Poissonian statistics or photon antibunching.

At the end of this section we turn to thermal effects considering briefly the interaction of the exciton-photon system with the reservoir at finite temperatures. We investigate how the temperature affects the Mandel factor and the correlation function of photon mode in comparison to the case of zero-temperature reservoir. The effects coming from thermal reservoirs are interesting for performing more realistic approach to generate nonclassical states in exciton-photon systems and for study of phenomena connecting quantum engineering and temperature.

The results for thermal photons in the range $n_{th} = 0.01 - 1$ are shown in Figs. 7(a),(b). Taking into account that $T = \hbar\omega_{th}/k_B$, for the frequencies of thermal photons $\omega_{th} = 10GHz$ the temperature in this range corresponds to $10^{-1}K - 10K$. In order to illustrate the difference between the case of zero-temperature resonator we assume here the parameters as in the previous case depicted in Fig. 3. As our calculation shows, the maximal values of mean photon numbers and the mean exciton numbers are approximately the same as in the case of pure resonator, while quantum statistics of oscillatory modes is changed due to the thermal noise. The light inside the cavity remains sub-Poissonian and anti-bunched for all time intervals, if $n_{th} = 0.05$. In this case the minimal values of Q-parameter and the correlation function remains nearly to the case of vacuum reservoir. As expected, the further increasing of the temperature leads to decreasing of quantum effects.

IV. CONCLUSIONS

In conclusion, we considered the quantum effects in a system consisting of an individual large quantum dot containing interacting excitons and single mode photonic cavity. We have shown that the nature of the external driving can substantially change the behavior of the system. Namely quantum effects become more pronounced in the case of the time modulated two mode driving as compared to the simple single mode external pump.

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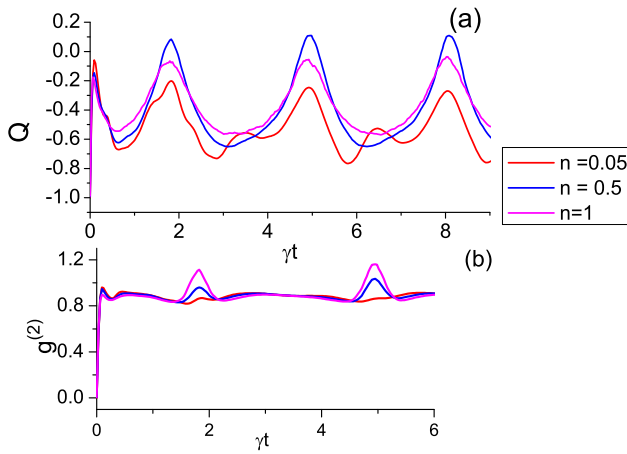


FIG. 7: Time evolution of the Mandel factor for three values of thermal photon numbers n_{th} (a). The autocorrelation function versus time-intervals for the several numbers of thermal photons (b). The parameters are as follows: $\Delta_{ph}/\gamma = 7.12$, $\Delta_{ex}/\gamma = 7.12$, $\chi/\gamma = 1$, $g/\gamma = 5$, $\Omega_1/\gamma = 5$, $\Omega_2/\gamma = 5$, $\delta/\gamma = 2$.

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